# Top–bottom mass hierarchy, $s - \mu$ puzzle and gauge coupling unification with split multiplets

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**Abstract.** Supersymmetric 5D $SU(5)$ grand unification is considered. $SU(5)$ is broken down to $G_{\mathrm{SM}} = SU(3) \times SU(2) \times U(1)$ by the $Z_2 \times Z_2'$ assignment of the bulk field(s). The matter fields are located at the fixed point(s). In the bulk, a Higgs multiplet $\bar{5}_H$ (containing the bottom doublet $H_1$) and the $SU(5)$ gauge multiplet are located. At one fixed point, $H_2$ (the top doublet) and the standard model matter multiplets are presented. Because of the difference of the locations of $H_1$ and $H_2$, one can obtain a hierarchy between top and bottom Yukawa couplings. We also present a possible way to understand the $s$–$\mu$ mass puzzle in this framework of the split multiplet.

## 1 Introduction

The unification of gauge coupling constants that goes under the name of grand unification is an attractive proposal [1], which cannot be understood in the standard model (SM). At the unification scale $M_{\mathrm{U}}$ the strong, weak and electromagnetic coupling constants are the same since they are described by a simple or semi-simple group $G$ for grand unification. Below the unification scale, this grand unification (GUT) group $G$ is broken down to the standard model group $SU(3) \times SU(2) \times U(1)$ and the difference between the SM couplings is generated [2]. Since the GUT unifies the interactions and some quarks and leptons are assigned in the same $G$ multiplet, the quark and lepton transition is possible in GUTs, triggering proton decay. The lepto-quark gauge bosons and colored scalars are responsible for proton decay. The lepto-quark gauge boson mass is the unification scale $M_{\mathrm{U}}$. But the Higgs multiplet containing the SM Higgs doublet must contain a light spectrum for the doublet. The colored partner of the doublet must be superheavy, or the proton lifetime is absurdly short, $< 10^{-9}$ s, for $O(1)$ couplings. For example, the $\bar{5}_H$ of $SU(5)$ contains a Higgs doublet field $H_1(Y = -1/2)$ and color triplet field $H_T(Y = 1/3)$, where it is assumed that $H_1$ is light and $H_T$ is superheavy. There occurs the difficulty of splitting the doublet–triplet masses, which is the split multiplet problem.

Recently, it was pointed out that the split multiplet problem can be understood in 5D theories with $S_1/Z_2 \times Z_2'$ orbifold compactification [3]. This is because of the geo-

metric twist of the gauge group such that some fields are projected out from the massless spectrum. Indeed, the orbifold compactification in string models [4] has shown already some models without colored scalars, realizing the split multiplet[1]. Thus, orbifold compactification in higher dimensional theories may be the underlying reason for the split multiplet [6–8]. In the context of this orbifold breaking of the GUT groups, some issues can be reconsidered such as the gauge coupling unification [9–13], the larger GUT groups [14], and the flavor unification [15,13] In this paper, we try to understand *geometrically* the top–bottom mass hierarchy and the $s$–$\mu$ puzzle [16].

The fifth dimensional coordinate $y$ is compactified to a circle $2\pi R \equiv 0$. Furthermore, the point $y = -a$ is identified with $y = a$ ($Z_2$ symmetry) and the point $y = (\pi R/2) + a$ is identified with $y = (\pi R/2) - a$ ($Z_2'$ symmetry). This modding introduces a fundamental region $y = [0, \pi R/2]$ and there arise two fixed points, $y = 0$ and $y = \pi R/2$. This geometry is used to twist the GUT multiplet. In particular the GUT multiplet $\bar{5}_H$ living in the bulk is twisted, the twisting being represented by $P = \mathrm{diag.}(1,1,1,1,1)$ and $P' = \mathrm{diag.}(-1,-1,-1,1,1)$. Obviously, the twisting breaks $SU(5)$. But in the bulk the $SU(5)$ symmetry is manifest above the unification scale and the gauge coupling unification is assumed above $M_{\mathrm{U}}$. The bulk fields are split into four different Kaluza–Klein (KK) categories $\phi_{i,j}$ with the $Z_2 \times Z_2'$ quantum numbers $(i, j)$,

$$\phi_{++} = \sum_{n=0}^{\infty} a_{2n} \phi_{++}^{(2n)}(x^{\mu}) \cos \frac{2ny}{R}, \tag{1}$$

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[1] The first model without colored scalars is Model 3 of [5]

$$\phi_{+-} = \sum_{n=0}^{\infty} a_{2n+1} \phi_{+-}^{(2n+1)}(x^\mu) \cos \frac{(2n+1)y}{R}, \qquad (2)$$

$$\phi_{-+} = \sum_{n=0}^{\infty} a_{2n+1} \phi_{-+}^{(2n+1)}(x^\mu) \sin \frac{(2n+1)y}{R}, \qquad (3)$$

$$\phi_{--} = \sum_{n=0}^{\infty} a_{2n+2} \phi_{--}^{(2n+2)}(x^\mu) \sin \frac{(2n+2)y}{R}, \qquad (4)$$

where $x^\mu$ is the 4D spacetime coordinate, $a_0 = (2/\pi R)^{1/2}$ and $a_n = (4/\pi R)^{1/2}$ for $n \neq 0$. The massless field is $\phi_{++}^{(2n)}$ for $n = 0$. In this way, the massless 4D Higgs doublet is obtainable from 5D while color triplets are all heavy.

Now let us extend this study to include $N = 1$ supersymmetry. In 5D, there exists an $N = 2$ supersymmetry. One $Z_2$ breaks down the $N = 2$ down to $N = 1$ and the other $Z_2$ breaks $G$ down to the SM. Two 4D spinors (e.g. two Weyl spinors) make up one 5D spinor. Thus, a 5D field theory is not anomalous. We can introduce only one $SU(5)$ fermion multiplet in the bulk without worrying about the anomaly, say a hypermultiplet $\bar{\mathbf{5}}_H$. Upon compactification, the $N = 1$ supermultiplets are

$$H_1^{(2n)} \left[ (++); \left( 1, 2, -\frac{1}{2} \right) \right], \text{ mass} = 2n/R, \qquad (5)$$

$$H_T^{(2n+1)} \left[ (+-); \left( 3, 1, \frac{1}{3} \right) \right], \text{ mass} = (2n+1)/R, \qquad (6)$$

$$\hat{H}_T^{(2n+1)} \left[ (-+); \left( \bar{3}, 1, -\frac{1}{3} \right) \right], \text{ mass} = (2n+1)/R, \qquad (7)$$

$$\hat{H}_D^{(2n+2)} \left[ (--); \left( 1, 2, \frac{1}{2} \right) \right], \text{ mass} = (2n+2)/R, \qquad (8)$$

where the brackets [] contain the quantum numbers of $Z_2 \times Z_2' \times SU(3) \times SU(2) \times U(1)$. The original 5D $SU(5)$ theory with one anti-quintet is anomaly free. But the orbifolding introduces one massless fermion doublet only, $H_1(n=0)$. The other massive fields in the bulk pair up to form massive KK towers of mass $m = n'/R$ where $n' = 1, 2, \cdots, \infty$. Since the low energy theory should be anomaly free, we are dictated to introduce brane fermions. So at one fixed point we introduce a 4D $N = 1$ SM Higgs supermultiplet $H_2$ with the quantum number $(1, 2, 1/2)$ under $SU(3) \times SU(2) \times U(1)$. At this field theory level, the introduction of anomaly canceling fermions at the fixed points is arbitrary. It is needed because of the renormalizability of the low energy effective theory. However, the orbifold compactification in string models introduces fixed point fermions definitely once the bulk fermions carry an anomaly [5,7].

Under the framework of the preceding paragraph, we will consider two models in Sect. 2.

(1) Model I: one $\bar{\mathbf{5}}_H$ in the bulk;
(2) Model II: one $\bar{\mathbf{5}}_H$ plus $(\bar{\mathbf{5}}_{f,1} + \bar{\mathbf{5}}_{f,2})$ in the bulk, where $H$ denotes a Higgs field and $f$ denotes some fermions of the SM.

In Sect. 3, we will try to understand the $s$–$\mu$ puzzle geometrically along the line of the split multiplet in the bulk, and present Model III for an explicit presentation.
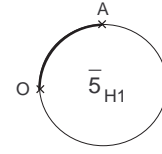


**Fig. 1.** $\bar{\mathbf{5}}_H$ is put in the bulk, and the SM fields and $H_2$ are located at the fixed point A

Since there the KK tower of the split multiplet appears in the bulk we expect a correction to $\alpha_s(M_Z)$ from the usual SUSY GUT prediction,

$$\delta\alpha_s(M_Z) \equiv \alpha_s^{\exp}(M_Z) - \alpha_s^{\text{SGUT},0}, \qquad (9)$$

which is $\delta\alpha_s(M_Z) = -0.013 \pm 0.0045$ [13]. The superscript 0 denotes that there is no threshold correction. We will show that in Models I, II and III the Kaluza–Klein mode corrections are in the favorable direction toward the experimental data.

## 2 Splitting $H_1$ and $H_2$ in the bulk and at a brane

At the minimal supersymmetric standard model (MSSM) level, $H_1$ (coupling to $b$ quark) and $H_2$ (coupling to $t$ quark) are not distinguished except for their gauge quantum numbers. Thus, the apparent disparity of the top–bottom masses is not understood. It is fixed either by a large top Yukawa coupling and a small bottom coupling with $\tan\beta \sim 1$ or by comparable Yukawa couplings and a large $\tan\beta$. In this section, we explore the possibility that the couplings and vacuum expectation values are comparable, but the mass hierarchy is understood in terms of a geometric origin [17]. Namely, the origin of $H_1$ and $H_2$ are different in a higher dimensional theory[2].

To concentrate on the $b$–$t$ disparity, we restrict our discussion to the third family only.

### 2.1 Model I

As the simplest model of the field theoretic orbifold compactification, let us introduce a $\bar{\mathbf{5}}_H$ in the 5D bulk. The compactification is $S_1/Z_2 \times Z_2'$ as shown in the Introduction. Because of the unification in the 5D bulk the gauge coupling is unified above the GUT scale $M_U$ which can be a string scale in a theory from string compactification. The $S_1/Z_2 \times Z_2'$ compactification produces one massless supermultiplet $H_1$ containing one Higgs doublet. The compactification is schematically drawn in Fig. 1 where two fixed points (3-branes) $O$ and $A$ are shown and the thick line is the fundamental region (= the bulk) in 5D. In the bulk $SU(5)$ gauge fields and $\bar{\mathbf{5}}_H$ live. At the 3-brane $A$ we locate

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[2] Without grand unification, separating $H_1$ and $H_2$ in the bulk and brane was considered before [18]. However, in our GUT theory, assigning $H_2$ at a brane is needed to explain the difference of $b$–$t$ mass scales in the low energy effective theory
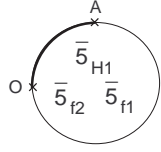
**Fig. 2.** Same as Fig. 1 except that the $(\nu_\tau, \tau)$ doublet and $b^c$ are put in the bulk

the missing Higgs doublet $H_2$ and the SM fields (including three copies of supermultiplets of 15 chiral fields). The 5D Lagrangian contains

$$S \supset \int \mathrm{d}^4x \int_0^{\pi R/2} \mathrm{d}y \left[ \partial^M H_1^\dagger(x,y) \partial_M H_1(x,y) \right.$$
$$\left. + \delta\left(y - \frac{\pi R}{2}\right) (\lambda_b H_1 Q D^c + f_t H_2 Q U^c) \right] \qquad (10)$$
$$= \int \mathrm{d}^4x \left[ \partial^\mu H_1^{(0)\dagger}(x) \partial_\mu H_1^{(0)}(x) + y_b H_1^{(0)} Q_3 D_3^c \right.$$
$$\left. + y_t H_2 Q_3 U_3^c \right], \qquad (11)$$

where $y_t = f_t$, $y_b = f_b(2/\pi M_U R)^{1/2}$, $\lambda_b = f_b/M_U^{1/2}$ and $H_1^{(0)}(x,y) = (2/\pi R)^{1/2} H_1^{(0)}(x)$. Thus, we obtain hierarchic masses:

$$\frac{m_b}{m_t} = \frac{1}{\tan\beta\sqrt{M_U R \pi/2}} \sim \frac{1}{60}. \qquad (12)$$

Note that the geometric suppression is the square root of $R$, which may not be large enough. Therefore, to enhance the suppression we consider the following model.

## 2.2 Model II

In Model I we inserted only $\bar{\mathbf{5}}_H$ in the bulk. Here, we introduce $b^c$ in the bulk also. The bulk field must be an $SU(5)$ multiplet. For no mass hierarchy between $b$ and $\tau$ masses, we need a complete multiplet. But an $SU(5)$ multiplet field in the bulk allows only a split massless field. For a complete multiplet $\bar{\mathbf{5}}$ to be massless, we have to introduce two $\bar{\mathbf{5}}$'s so that an anti-quark singlet from one $\bar{\mathbf{5}}$ and a lepton doublet from the other $\bar{\mathbf{5}}$ survives as a massless field by appropriately twisting the bulk fields. In Fig. 2, we assign the fields in the bulk and at the 3-brane A. Except the quintets containing $b^c$, $\tau_L$, all the SM fermions are located at $A$. Of course, we locate $H_2$ at $A$ to cancel the gauge anomaly. The relevant 5D Lagrangian is

$$S \supset \int \mathrm{d}^4x \int_0^{\pi R/2} \mathrm{d}y \left[ \partial^M H_1^\dagger(x,y) \partial_M H_1(x,y) \right.$$
$$+ \bar{D}_3^c(x,y) i \partial_M \gamma^M D_3^c(x,y) + \bar{L}_3(x,y) i \partial_M \gamma^M L_3(x,y)$$
$$\left. + \delta\left(y - \frac{\pi R}{2}\right) \right.$$

$$\times \left. (\lambda_b H_1 Q_3 D_3^c + f_t H_2 Q_3 U_3^c + \lambda_\tau H_1 L_3' E_3^c) \right]$$
$$= \int \mathrm{d}^4x \left[ \cdots + y_b H_1^{(0)} Q_3 D_3^{c(0)} + y_t H_2 Q_3 U_3^c \right.$$
$$\left. + y_\tau H_1^{(0)} L_3'^{(0)} E_3^c \right], \qquad (13)$$

from which we obtain a linear relation in $R$: $y_b \sim y_\tau \sim (\pi M_U R/2)^{-1} y_t$. Note that $\lambda_b \sim f_b/M_U$, $\lambda_\tau \sim f_\tau/M_U$, $H_1^{(0)}(x,y) = (2/\pi R)^{1/2} H_1^{(0)}(x)$, $D_3^{c(0)}(x,y) = (2/\pi R)^{1/2} D^{c(0)}(x)$ and $L_3'^{(0)}(x,y) = (2/\pi R)^{1/2} L_3'^{(0)}(x)$.

## 2.3 Running of gauge coupling constants

The mass scales of interest in our scenario are the electroweak scale, the unification scale (or the string scale) $M_U$, and the inverse compactification length $M_c = 1/R$. We assume that the compactification mass is smaller than the unification mass so that the running of the Kaluza–Klein (KK) towers between $M_U$ and $M_c$ helps for the unification condition. Let us define the ratio of these two scales as $2N$,

$$N = \frac{M_U}{2M_c}. \qquad (14)$$

The masses of the KK modes are

$$\begin{aligned}
(+,+) &: 2nM_c & b_i \; (b_i^0 \text{ for } n=0) \\
(+,-) &: (2n+1)M_c \; c_i \\
(-,+) &: (2n+1)M_c \; \bar{c}_i \\
(-,-) &: (2n+2)M_c \; \bar{b}_i,
\end{aligned} \qquad (15)$$

where the columns show $(P, P')$ quantum numbers, KK masses, and the $\beta$ function coefficients. The tower of KK excitations up to $M_U$ contributes to the running of gauge couplings at their thresholds [19].

At the scale $\mu$ below the compactification scale, the gauge coupling constant is

$$\frac{8\pi^2}{g_i^2(\mu)} = \frac{8\pi^2}{g_U^2} + b_i' \ln\frac{M_c}{\mu} + b_i' \ln(2N) + (b_i + \bar{b}_i)$$
$$\times \sum_{n=1}^N \ln\frac{2N}{2n} + (c_i + \bar{c}_i) \sum_{n=1}^N \ln\frac{2N}{2n-1}, \qquad (16)$$

up to a threshold correction $\Delta_i$. $g_U$ is the unification coupling. Stirling's formula gives $\sum_{n=1}^N \ln(2N/2n) \simeq N - (1/2)\ln(2\pi N)$ and

$$\sum_{n=1}^N \ln((2N)/(2n-1)) \simeq N - (1/2)\ln 2 \simeq N.$$

Thus, the low energy MSSM couplings become

$$\frac{8\pi^2}{g_i^2(\mu)} = \frac{8\pi^2}{g_U^2} + b_i' \ln\frac{M_c'}{\mu} + \tilde{b}' \ln\frac{M_U}{M_c'} + \frac{b}{2}\left[\frac{M_U}{M_c} - 1\right]$$
$$+ \tilde{b}_i \ln\frac{M_U}{M_c'}, \qquad (17)$$

where $M'_c = M_c/\pi$, and

$$b \equiv b_i + c_i + \bar{b}_i + \bar{c}_i, \quad \text{for all } i,$$
$$\tilde{b}_i \equiv b_i^0 - \frac{1}{2}(b_i + \bar{b}_i),$$
$$b'_i = (33/5, 1, -3), \quad \text{in MSSM},$$
$$\tilde{b}' = b'_i - b_i^0, \quad \text{for all } i. \tag{18}$$

*Model I:* In Model I, from the fields $\bar{\mathbf{5}}_H$ in the bulk we obtain

$$b_i^H = 2b_i^{H_0}, \quad \tilde{b}_i = b_i^{H_0} - \frac{1}{2}b_i^H = 0, \tag{19}$$

and from the field $H_2$ in the brane

$$\tilde{b}_i : (\tilde{b}_3, \tilde{b}_2, \tilde{b}_1)^{H_2} = \left(0, \frac{1}{2}, \frac{3}{10}\right). \tag{20}$$

From the vector multiplet in the bulk, $b_i^A = (2/3)b_i^{A_0}$,

$$\tilde{b}_i = b_i^{A_0} - \frac{1}{2}b_i^A : (\tilde{b}_3, \tilde{b}_2, \tilde{b}_1)^V = (-6, -4, 0). \tag{21}$$

The sum of the brane Higgs and the bulk vector contributions define the total value, $\tilde{b}_i = \tilde{b}_i^{H_2} + \tilde{b}_i^V$. Therefore, from (17), we obtain a relation between the couplings at the electroweak scale

$$\frac{1}{g_3^2} = \frac{12}{7}\frac{1}{g_2^2} - \frac{5}{7}\frac{1}{g_1^2} + \frac{\tilde{b}}{8\pi^2}\ln\frac{M_U}{M'_c}, \tag{22}$$

where $\tilde{b} = \tilde{b}_3 - (12/7)\tilde{b}_2 + (5/7)\tilde{b}_1 = 3/14$.

The strong coupling unification considered in [13] is due to the duplication of matter fields appearing in the extension of chiral multiplets to hypermultiplets which make the gauge coupling strong at high energy scales. But in all models considered here, most matter fields are living at the brane and the 5D gauge theory becomes asymptotically free: $b = -9, -7, -6$ in (17) for Models I, II, and III, respectively.

In the unification models, such as in SUSY $SU(5)$, one can determine the unification mass and gauge coupling constant $\alpha_U$ at the unification scale. Namely, if $M_U$ and $\alpha_U$ are given, one can predict $\alpha_i$ at the electroweak scale. These coupling constants satisfy the relation given in (22), and the experimental values at $M_Z$ may not satisfy (22). In our models, the onset of the KK modes introduces another parameter $M_c$. Thus, we can satisfy the condition (22) by appropriately choosing $M_c$. However, in our split multiplet models the unified gauge coupling constant is extremely small due to the asymptotic freedom. Namely, $M_c$ turns out to be far below the string scale, $M'_c = 4.5 \times 10^9$ GeV with $M_U = 2.8 \times 10^{19}$ GeV.

However, it may be a better treatment of the problem if we satisfy one condition. We choose the condition by the ratio $M_U/M'_c$ in view of the experimental errors including the error in $\alpha_s$ and other effects (running effects due to Yukawa couplings and two loop runnings). Thus, the unification condition (22) is satisfied approximately but not exactly due to the error bars allowed and hence

we will study just the modification of $\alpha_s$. Then, $\delta\alpha_s = \alpha_s^{\exp}(M_Z) - \alpha_s^{\text{SGUT}}$, where $\alpha_s^{\text{SGUT}} = \alpha_s^{\text{SGUT},0} + \Delta_{\text{KK}}\alpha_s$ is the KK mode corrected value in [13] with reasonable choices of the ratio $M_U/M'_c$. For $M_U/M'_c = 10^2, 10^3, 10^4$, we obtain $\Delta_{\text{KK}}\alpha_s \simeq -0.003, -0.004, -0.005$, respectively. Then, $\delta\alpha_s = (-0.010, -0.009, -0.008) \pm 0.0045$ which correspond to $2.3\sigma, 2.0\sigma, 1.8\sigma$ away from the experimental data. In all cases considered above the additional logarithmic running reduces the discrepancy between experimental value and the prediction of SUSY GUT even with no threshold correction. The coupling at the unification scale becomes $\alpha_U = 2 \times 10^{-2}, 4 \times 10^{-3}$ and $4 \times 10^{-4}$, respectively. We used the conventional value $\alpha_s \simeq 1/24$ at $\mu = M_c$. At $M_U$ we also cut off the power running and there appears an $O(1)$ uncertainty in $N$, which however does not affect the unification condition.

*Model II:* We can repeat the same calculation for Model II. But note that the additional fields $\bar{\mathbf{5}}_{f,1}$ and $\bar{\mathbf{5}}_{f,2}$ have the following KK modes

$$\bar{\mathbf{5}}_{f,1} = ((++), (+-), (-+), (--)) = (D_3^c, L_3, \hat{L}_3, \hat{D}_3^c),$$
$$\bar{\mathbf{5}}_{f,2} = ((+-), (++), (--), (-+))$$
$$= (D_3'^c, L'_3, \hat{L}'_3, \hat{D}_3'^c), \tag{23}$$

so that the zero modes are $(D_3^c, L'_3)_{n=0}$ which mimicks a GUT multiplet. Other massive GUT-like multiplets are the even KK modes $(++) = (D_3^c, L'_3)_{n\neq0} = \bar{\mathbf{5}}$ and $(--) = (\hat{D}_3^c, \hat{L}'_3) = \mathbf{5}$ which contribute to the log running. The odd KK modes contributes only to the power running. Thus, the difference of gauge couplings and the ratio $N$ are not changed, viz. (22). For $\delta\alpha_s = 0$ the unification coupling is changed to $\alpha_U \simeq 1 \times 10^{-9}$ for the exact unification. A similar analysis as in the study of Model I for $M_U/M'_c = 10^2, 10^3$, and $10^4$, the coupling constant at the unification scale becomes $\alpha_U \simeq 2 \times 10^{-2}, 5 \times 10^{-3}$, and $5 \times 10^{-4}$, respectively. The KK mode correction to $\delta\alpha_s$ is the same as in Model I.

# 3 Splitting the second family fermions in the bulk and at a brane

As discussed in the preceding section, there are a lot of possibilities for obtaining hierarchies of couplings by locating some fields in the bulk and some fields at a brane. In this section, we explore one more possibility for geometrically generating hierarchical coupling structure. One of the puzzles in the $SU(5)$ GUT is that in the second family the quark Yukawa coupling is too small (by a factor of 3) compared to the lepton coupling, which is the $s$–$\mu$ puzzle. To obtain a desired suppression for the $s$ quark coupling, Georgi and Jarlskog introduced $\mathbf{45}_H$ in addition to the usual $\mathbf{5}_H$ [16]. In our scenario of keeping a split part of an $SU(5)$ multiplet as a massless spectrum in the bulk, there is a possibility of geometrically understanding the $s$–$\mu$ puzzle.

For simplicity, we modify the simplest example, Model I of the previous section, and comment on another possibility after the discussion on Model III. The Higgs fields,
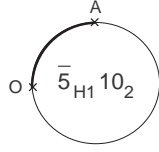
**Fig. 3.** Same as Fig. 1 except that the $(c, s)$ doublet is put in the bulk

the first and the third family members are the same as in Model I. We only change the members of the second family.

*Model III:* Among the second family members, some fields are put in the bulk. It is a split multiplet from **10**. In the bulk, the members of $\mathbf{10} \equiv (Q_2, U_2^c, E_2^c, \hat{U}_2^c, \hat{E}_2^c, \hat{Q}_2) = [(3,2), (\bar{3},1), (1,1), (3,1), (1,1), (\bar{3},2)]$ under $SU(3) \times SU(2)$ are assigned the $Z_2 \times Z_2'$ parity as $(++), (+-),$ $(+-), (-+), (-+),$ and $(--)$, respectively. Thus, only the quark doublet $Q_2$ has a zero mode spectrum $Q_2^{(0)}$ which we interpret as the second family quark doublet. At low energy, the theory must be anomaly free and hence we locate the rest members of the second family, $s^c, c^c, \mu^c, L_2 = (\nu_\mu, \mu)_L$, at the brane, which is shown in Fig. 3. The 5D Lagrangian contains

$$S \supset \int \mathrm{d}^4x \int_0^{\pi R/2} \left[ \partial^M H_1^\dagger(x,y)\partial_M H_1(x,y) \right.$$
$$+ \bar{Q}_2(x,y)\mathrm{i}\partial_M \gamma^M Q_2(x,y)$$
$$+ \delta\left( y - \frac{\pi R}{2} \right)$$
$$\times (\lambda_s H_1 Q_2 D_2^c + \lambda_c H_2 Q_2 U_2^c + \lambda_\mu H_1 L_2 \mu^c + \lambda_b H_1 Q_3 D_3^c$$
$$\left. + f_t H_2 Q_3 U_3^c + \cdots)] \right. \tag{24}$$
$$= \int \mathrm{d}^4x \left[ y_s H_1^{(0)} Q_2^{(0)} D_2^c + y_\mu H_1^{(0)} L_2 E_2^c + \cdots \right].$$

Note that $\lambda_s \sim f_s/M_U$, $\lambda_\mu \sim f_\mu/\sqrt{M_U}$, $y_s = (2/\pi M_U R) f_s$, $y_\mu = (2/\pi M_U R)^{1/2} f_\mu$, implying

$$\frac{y_s}{y_\mu} \sim \frac{1}{\sqrt{M_U R}}. \tag{25}$$

Thus, the strange quark Yukawa coupling is suppressed compared to the muon Yukawa coupling.

From the bulk zero mode $Q_2^{(0)}$, $\tilde{b}_i = b_{Q_2}^{(0)} - (1/2)(b_{Q_2} + b_{\hat{Q}_2}) = 0$. From the brane fields, $U_2^c$ and $E_2^c$, $\tilde{b}_i = (b_{U^c})_i + (b_{E^c})_i = (1/2, 0, 4/5) + (0, 0, 3/5) = (1/2, 0, 7/5)$ for $i = 3, 2, 1$. From the brane Higgs $H_2$, $\tilde{b}_i = (0, 1/2, 3/10)$. From the bulk Higgs $\bar{5}_{H_1}$, $\tilde{b}_i = 0$. From the vector multiplet, $\tilde{b}_i = (-6, -4, 0)$. Thus, we obtain

$$\tilde{b}_i = \left( -\frac{11}{2}, -\frac{7}{2}, \frac{17}{10} \right), \quad \text{for } i = 3, 2, 1. \tag{26}$$

Therefore,

$$\tilde{b} = \tilde{b}_3 - \frac{12}{7}\tilde{b}_2 + \frac{5}{7}\tilde{b}_1 = \frac{12}{7}. \tag{27}$$

Model III is interesting since it turns out that $M_c'$ is very large, $\simeq 1.5 \times 10^{15}$ GeV, and $M_U$ is the usual unification scale $2.5 \times 10^{16}$ GeV in order to satisfy (22). The unification coupling constant is also close to the SGUT value $\alpha_U \simeq 0.03 \simeq 1/30$. Due to the large $\tilde{b}$ compared to the other models, Model III allows for the perfect unification with the logarithmic running between a small scale difference of $M_U/M_c' = 10^{1.22}$.

If we consider Model III with the backbone of Model II instead the backbone of Model I, we obtain a better relation between the top–bottom mass hierarchy, since the suppression will be linear in $1/R$. Assuming all the couplings to be order 1, we obtain $m_\mu/m_s \sim (m_t/m_b \cdot \tan\beta)^{1/2} = (60/\tan\beta)^{1/2} \sim 3$ at the unification scale, implying $\tan\beta \sim 7$. $\delta\alpha_s$ is the same as in Model III.

# 4 Conclusion

In this paper, we studied a new possibility for a field theoretic orbifold compactification possessing supersymmetry, which was applied to the top–bottom mass hierarchy and the $s$–$\mu$ puzzle. This possibility relies on the missing massless spectrum in the bulk. The 5D bulk theory with any fermion representation is anomaly free, but the orbifold compactification may project out split multiplet in the bulk. This situation has been observed in orbifold compactifications in string models [5,7]. Because of the split multiplet, the anomaly-free condition dictates us to put some massless fermions at the brane so that the resulting 4D theory is anomaly free. In string examples, the field content and assignment of the fields at the fixed points are determined uniquely by the modular invariance requirement. But in our field theoretic example, the field content and the location are arbitrary. In this paper, we chose the simplest possibility.

In our examples, we put the $\bar{5}_H$ (containing the $H_1$ Higgs doublet) in the bulk. Because of the twisting, only $H_1$ from the $\bar{5}_H$ remains massless in the bulk. Thus, the needed $H_2$ is put at a brane where the SM fields are located. Thus, the bottom and the top quarks have geometrically different factors for the effective 4D Yukawa couplings, rendering a top–bottom mass hierarchy. To enhance the hierarchical factor some SM fermions are put in the bulk in our second example. Similarly, the $s$–$\mu$ puzzle is understood geometrically by putting the strange quark doublet in the bulk, thus reducing the strange quark Yukawa coupling compared to the muon Yukawa coupling. There are other applications along this line, e.g. reducing the up quark mass compared to the down quark mass. In all the examples we considered, the corrections to the strong coupling constant are in the right direction, making the low energy effective MSSM predictions closer to the experimental value.

One tempting question to ask in this scenario might be the $\mu$ problem [20]. Certainly, one cannot write $\mu H_1 H_2$ in the bulk. It can be written only at the brane A. But the need to introduce $H_2$ at A is below the compactification scale $M_c = 1/R$. Therefore, a dimensional parameter such

as $\mu$ must have a suppression factor, certainly less than $M_c$. But at this moment, we do not understand geometrically how large the suppression factor is. We may need an additional discrete or global symmetry to sufficiently suppress $\mu$. In any case, these extra symmetries are needed for proton longevity.

The field theoretic orbifolding considered recently is very simple compared to the string theory orbifolding. However, it seems to be arbitrary in choosing and assigning the fields, and we expect that some string compactification in the future may lead to the above types of field theoretic orbifolding so that the assignment of the $H_2$ and the SM fields at the brane is no longer arbitrary.

# References

1. H. Georgi, S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1973); J.C. Pati, Abdus Salam, Phys. Rev. D **8**, 1240 (1973); Phys. Rev. D **10**, 275 (1974)
2. H. Georgi, H.R. Quinn, S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974)
3. Y. Kawamura, Prog. Theor. Phys. **103**, 613 (2000) [hep-ph/9902423]; hep-ph/0012125; hep-ph/0012352; G. Altarelli, F. Ferugulio, Phys. Lett. B **511**, 257 (2001) [hep-ph/0102301]
4. L. Dixon, J. Harvey, C. Vafa, E. Witten, Nucl. Phys. B **261**, 651 (1985); Nucl. Phys. B **274**, 285 (1986)
5. L. Ibanez, J.E. Kim, H.P. Nilles, F. Quevedo, Phys. Lett. B **191**, 282 (1987)
6. L.E. Ibanez, J. Mas, H.P. Nilles, F. Quevedo, Nucl. Phys. B **301**, 157 (1988); J.E. Kim, Phys. Lett. B **207**, 434 (1988); J.A. Casas, C. Munoz, Phys. Lett. B **214**, 63 (1988); E.J. Chun, J.E. Kim, H.P. Nilles, Nucl. Phys. B **370**, 105 (1992)
7. J.E. Kim, Phys. Lett. B **207**, 434 (1988)
8. H.B. Kim, J.E. Kim, Phys. Lett. B **300**, 343 (1993)
9. L.J. Hall, Y. Nomura, Phys. Rev. D **64**, 055003 (2001) [hep-ph/0103125]; Y. Nomura, D.R. Smith, N. Weiner, Nucl. Phys. B **613**, 417 (2001) [hep-ph/0104041]
10. A. Hebecker, J. March-Russel, Nucl. Phys. B **613**, 3 (2001) [hep-ph/0106166]
11. L.J. Hall, Y. Nomura, D.R. Smith, hep-ph/0107331
12. R. Contino, L. Pilo, R. Rattazzi, Trincherini, hep-ph/0108102
13. L.J. Hall, Y. Nomura, hep-ph/0111068
14. L.J. Hall, Y. Nomura, T. Okui, D.R. Smith, hep-ph/0108071; T. Asaka, W. Buchmüller, L. Covi, hep-ph/0108021; T. Li, hep-ph/0108120; R. Dermisek, A. Mafi, hep-ph/0108139
15. L.J. Hall, J. March-Russel, T. Okui, D.R. Smith, hep-ph/0108161
16. H. Georgi, C. Jarlskog, Phys. Lett. B **86**, 297 (1979)
17. N. Arkani-Hamed, S. Dimopoulos, G. Dvali, J. March-Russel, hep-ph/9811448; K.R. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. B **557**, 25 (1999) [hep-ph/9811428]
18. N. Arkani-Hamed, M. Schmaltz, Phys. Rev. D **61**, 033055 (2000) [hep-ph/9903417]; N. Arkani-Hamed, L. Hall, D. Smith, N. Weiner, Phys. Rev. D **61**, 116003 (2000) [hep-ph/9909326]; E.A. Mirabelli, M. Schmaltz, Phys. Rev. D **61**, 113011 (2000) [hep-ph/9912265]; G. Dvali, M. Shifman, Phys. Lett. B **475**, 295 (2000) [hep-ph/0001072]; D.E. Kaplan, T.M. Tait, JHEP **0006**, 020 (2000) [hep-ph/0004200]; D.E. Kaplan, T.M. Tait, JHEP **0111**, 051 (2001) [hep-ph/0110126]; M. Kakizaki, M. Yamaguchi, hep-ph/0110266
19. K.R. Dienes, E. Dudas, T. Gherghetta, Phys. Lett. B **436**, 436 (1998); Nucl. Phys. B **537**, 47 (1999)
20. J.E. Kim, H.P. Nilles, Phys. Lett. B **138**, 150 (1984)
21. C.A. Scrucca, M. Serone, L. Silverstrini, F. Zwirner, Phys. Lett. B **525**, 169 (2002) [hep-th/0110073]
22. N. Arkani-Hamed, A.G. Cohen, H. Georgi, Phys. Lett. B **516**, 395 (2001) [hep-th/0103135]
23. C.G. Callan, J.A. Harvey, Nucl. Phys. B **250**, 427 (1985)

**Note added in proof:** Recently, there appeared an argument [21] that it would not be possible to have a consistent SUSY field theory on the $S^1/(Z_2 \times Z_2')$ orbifold with a single bulk Higgs multiplet, since there are gauge anomalies localized at the orbifold fixed points [22]. In our case, however, the local gauge anomalies can be canceled by introducing a brane Higgs field and 5D Chern–Simons terms in the bulk [23,22]. The models considered in the present paper are consistent up to introducing the Chern–Simons terms.